Math 13 – Liberal Arts Math – HW4 – Spring '13 **Name________________**

Write all responses on separate paper. Show your work for credit. Write in complete sentences.

1. Graph the two lines described by the equations $2x + 7y = 61$ and $3x + 4y = 46$ and show the coordinates of the point of intersection for the lines represented by the equations.

2. With the given the constraints for the following linear programming mixture problem, graph the feasible region. $2x + 3y \le 1800$
   $x \ge 0; y \ge 0$
3. With the given the constraints for the following linear programming mixture problem, graph the feasible region. $2x + 3y \le 180$
   $5x + 2y \le 230$
   $x \ge 0; y \ge 0$
4. With the given the constraints for the following linear programming mixture problem, graph the feasible region. $x + 3y \le 23$
   $3x + 7y \le 50$
   $x \ge 2; y \ge 4$
5. Find the constraint inequalities and the profit formula for this linear programming mixture problem: Toni has a small business producing dried floral wreaths and table arrangements. Each wreath takes 7 hours to produce, uses 12 stems of flowers, and earns a profit of $23. Each table arrangement takes 5 hours to produce, uses 20 stems of flowers, and earns a profit of $26. Toni can work no more than 30 hours per week and has a steady supply of 100 stems of dried flowers per week. What should Toni's production schedule be to optimize profit?
6. Write the constraint inequalities and the profit formula for this linear programming mixture problem: Amazin' Raisin Baking Co. makes both raisin cake and raisin pie. A batch of raisin cakes requires 5 lbs. of flour, 2 lbs. of sugar, and 1 lb. of raisins. A batch of raisin pies requires 2 lbs. of flour, 3 lbs. of sugar, and 4 lbs. of raisins. There are 165 lbs. of flour, 110 lbs. of sugar, and 120 lbs. of raisins available each week. Standing orders require at least five batches of raisin cakes and eight batches of raisin pies per week. If profit on a batch of raisin cakes is $35 and profit on a batch of raisin pies is $40, how many batches of each should be made per week to maximize profit?
7. Solve this linear programming mixture problem: A small stereo manufacturer makes a receiver and a CD player. Each receiver takes 8 hours to assemble, 1 hour to test and ship, and earns a profit of $30. Each CD player takes 15 hours to assemble, 2 hours to test and ship, and earns a profit of $50. There are 160 hours available in the assembly department and 26 hours available in the testing and shipping department. What should the production schedule be to maximize profit?
8. Find the maximum value of P, where P = 3x + 4y subject to the constraints.
   $$x \ge 0;\ y \ge 0;\ x + 2y \le 8;\ x + y \le 5$$
9. Find the maximum value of P, where P = 10x + 100y subject to the constraints.
   $$x \ge 0;\ 2x + 5y \le 20;\ 2x + y \le 12$$
10. Solve this linear programming mixture problem: Kim and Lynn produce pottery vases and bowls. A vase requires 25 oz. of clay and 5 oz. of glaze. A bowl requires 20 oz. of clay and 10 oz. of glaze. There are 500 oz. of clay available and 160 oz. of glaze available. The profit on one vase is $5 and the profit on one bowl is $3.

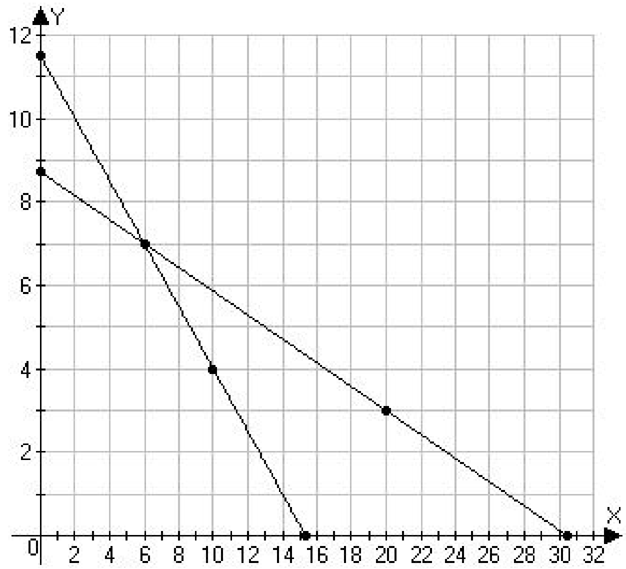
Math 13 – Liberal Arts Math – HW4 – Solutions.

1. Graph the two lines described by the equations $2x + 7y = 61$ and $3x + 4y = 46$ and show the coordinates of the point of intersection for the lines represented by the equations.

$2x+7y=61$			$3x+4y=46$	
x	y		x	y
0	$\frac{65}{7} \approx 8.7$		0	11.5
6	7		6	7
20	3		10	4
30.5	0		15.3	0

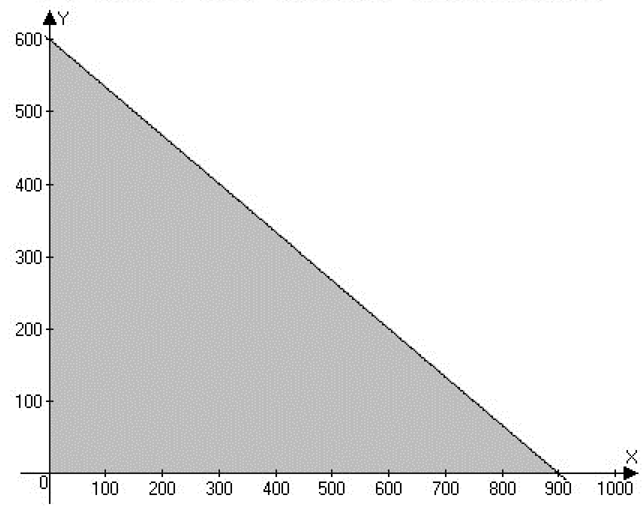
The solution is $(x, y) = (6,7)$

SOLN:



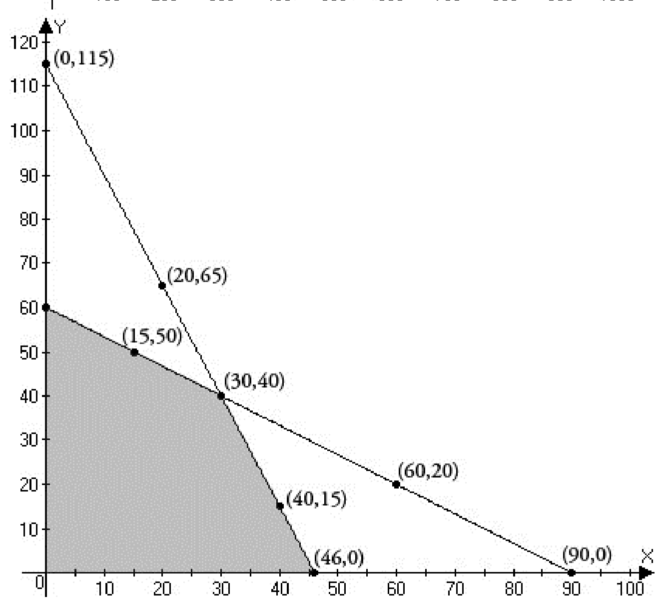
2. With the given the constraints for the following linear programming mixture problem, graph the feasible region.
$2x + 3y \leq 1800$
$x \geq 0; y \geq 0$

The inequalities $x \geq 0; y \geq 0$ restrict x and y to the first quadrant while
$$2x + 3y \leq 1800$$
Ensures we are below the line with intercepts at (0,600) and (900,0).



3. With the given the constraints for the following linear programming mixture problem, graph the feasible region.
$2x + 3y \leq 180$
$5x + 2y \leq 230$
$x \geq 0; y \geq 0$
SOLN:

$2x + 3y = 180$			$5x + 2y = 230$	
x	y		x	y
0	60		0	115
15	50		20	65
30	40		30	40
60	20		40	15
90	0		46	0

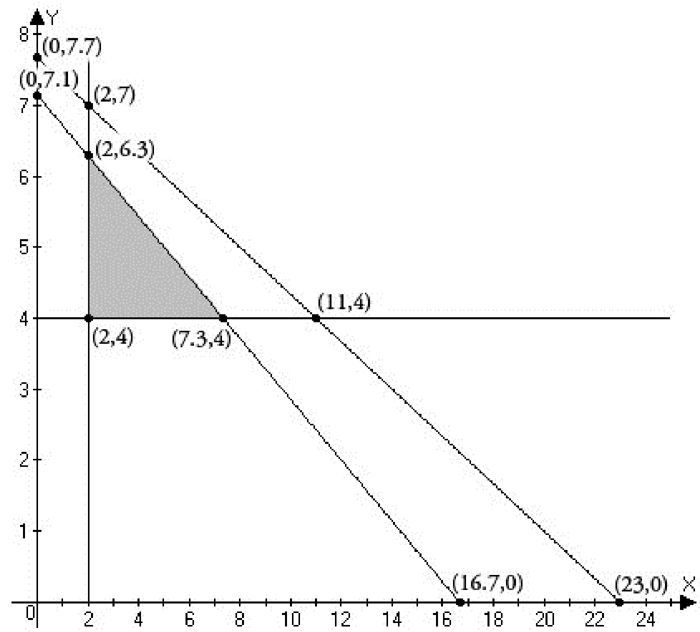


4. With the given the constraints for the following linear programming mixture problem, graph the feas ible region.

$$x + 3y \leq 23$$
$$3x + 7y \leq 50$$
$$x \geq 2; y \geq 4$$

Tabulate points on boundaries.

$x + 3y = 23$		$3x + 7y = 50$	
x	y	x	y
0	$\frac{23}{3} \approx 7.7$	0	$\frac{50}{7} \approx 7.1$
2	7	2	$\frac{44}{7} \approx 6.3$
11	4	$\frac{22}{3} \approx 7.3$	4
23	0	$\frac{50}{3} \approx 16.7$	0



The feasible region is shaded.

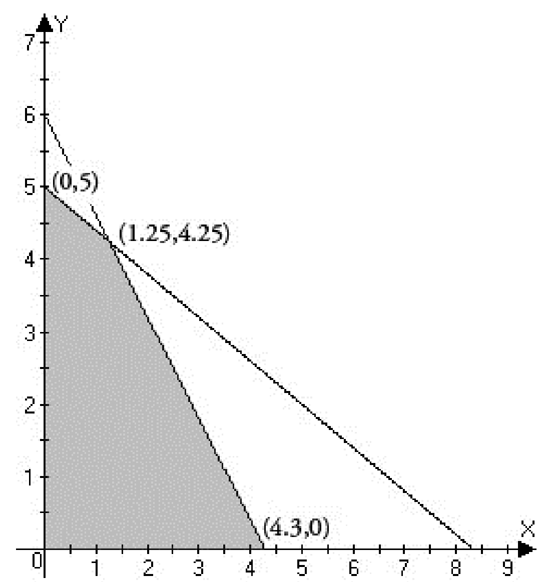
5. Find the constraint inequalities and the profit formula for this linear programming mixture problem: Toni has a small business producing dried floral wreaths and table arrangements. Each wreath takes 7 hours to produce, uses 12 stems of flowers, and earns a profit of $23. Each table arrangement takes 5 hours to produce, uses 20 stems of flowers, and earns a profit of $26. Toni can work no more than 30 hours per week and has a steady supply of 100 stems of dried flowers per week. What should Toni's production schedule be to optimize profit?

SOLN: Let x = the number of wreaths produced and y = the number of table arrangements produced.
Then the LP problem is to
Maximize $P = 23x + 26y$
Subject to: $7x + 5y \leq 30$
$$12x + 20y \leq 100$$
$$x \geq 0;\ \ y \geq 0$$

From the graph we see the feasible region includes the intercepts (0,5) and (25/3,0), but also the intersection of the two lines, which we can find by subtracting ¼ the second boundary line from the first to get $4x = 5$ or $x = 1.25$, which when substituted back yields $y = 4.25$.

$$P(0,5) = 130$$
$$P(1.25,4.25) = 139.25$$
$$P\left(\frac{30}{7},0\right) \approx 98.67$$

So the maximum value occurs at (1.25,4.25)



6. Write the constraint inequalities and the profit formula for this linear programming mixture problem: Amazin' Raisin Baking Co. makes both raisin cake and raisin pie. A batch of raisin cakes requires 5 lbs. of flour, 2 lbs. of sugar, and 1 lb. of raisins. A batch of raisin pies requires 2 lbs. of flour, 3 lbs. of sugar, and 4 lbs. of raisins. There are 165 lbs. of flour, 110 lbs. of sugar, and 120 lbs. of raisins available each week. Standing orders require at least five batches of raisin cakes and eight batches

of raisin pies per week. If profit on a batch of raisin cakes is \$35 and profit on a batch of raisin pies is \$40, how many batches of each should be made per week to maximize profit?

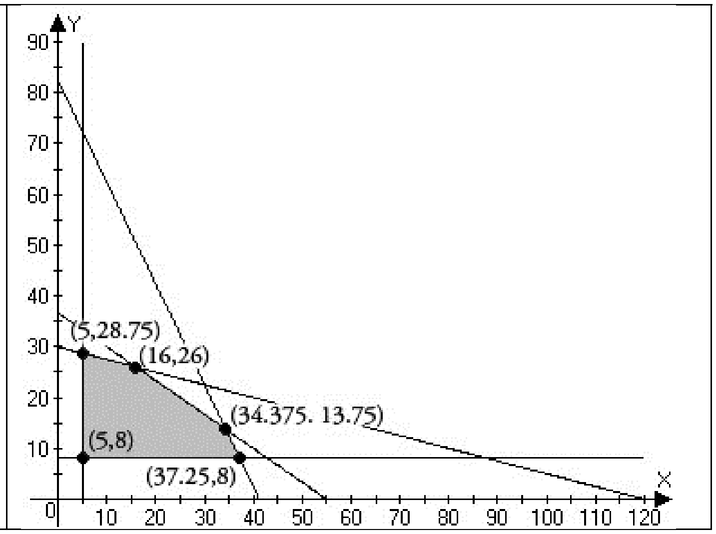
SOLN: Let x = the number of cakes and y = the number of pies. Then the LP problem is to

$$\text{Maximize } P = 35x + 40y$$
$$\text{Subject to: } 4x + 2y \le 165$$
$$2x + 3y \le 110$$
$$x + 4y \le 120$$
$$x \ge 5; y \ge 8$$

The object evaluated at the corners:

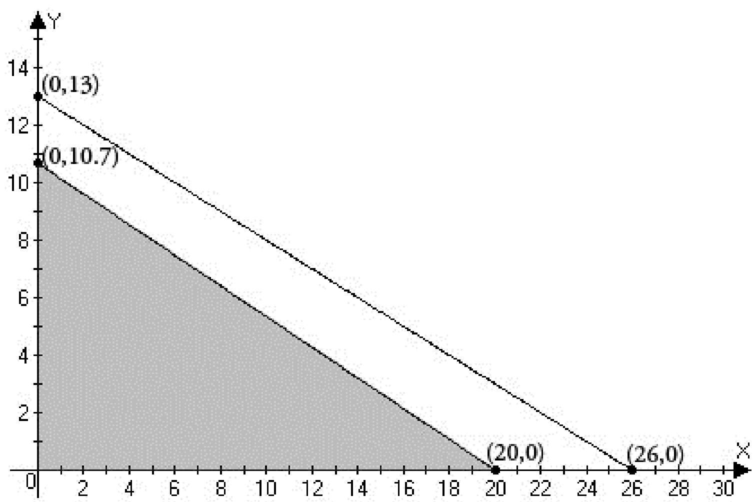
$$P(5,28.75) = 1325$$
$$P(16,26) = 1600$$
$$P(34.375,13.75) = 1753.125$$
$$P(37.25,8) = 1623.75$$

So the max is at $(34.375,13.75)$.



7. Solve this linear programming mixture problem: A small stereo manufacturer makes a receiver and a CD player. Each receiver takes 8 hours to assemble, 1 hour to test and ship, and earns a profit of \$30. Each CD player takes 15 hours to assemble, 2 hours to test and ship, and earns a profit of \$50. There are 160 hours available in the assembly department and 26 hours available in the testing and shipping department. What should the production schedule be to maximize profit?
   SOLN: Let x = the number of receivers and y = the number of CD players.
   Then the LP problem is to Maximize $P = 30x + 50y$

$$\text{subject to: } 8x + 15y \le 160$$
$$x + 2y \le 26$$
$$x \ge 0; \; y \ge 0$$



Evidently, the second constraint is redundant and the corners of the feasible region are $(0,0), (0, 10.\overline{6}), (20,6)$ and P is maximized at (20,6) where P = 600. So they should make 20 receivers and no CD players. Unless, of course, somebody really wants a CD player.

8. Find the maximum value of P, where $P = 3x + 4y$ subject to the constraints.

$$x \ge 0; \; y \ge 0; \; x + 2y \le 8; \; x + y \le 5$$

SOLN: The maximum value of P occurs at $(x, y) = (2,3)$ where P = 18.

9. Find the maximum value of P, where $P = 10x + 100y$ subject to the constraints.

$$x \geq 0;\ 2x + 5y \leq 20;\ 2x + y \leq 12$$

SOLN: The maximum occurs at the point (6, 0), where P = 600.

10. Solve this linear programming mixture problem: Kim and Lynn produce pottery vases and bowls. A vase requires 25 oz. of clay and 5 oz. of glaze. A bowl requires 20 oz. of clay and 10 oz. of glaze. There are 500 oz. of clay available and 160 oz. of glaze available. The profit on one vase is \$5 and the profit on one bowl is \$3.

SOLN: Let x = the number of vases and y = the number of bowls.

The the LP problem is to maximize $P = 5x + 3y$ subject to: $25x + 20y \leq 500$

$$5x + 10y \leq 160$$

$$x \geq 0; y \geq 0$$

The object function evaluated at the corners of the feasible region are $P(0,0) = 0$, $P(20,0) = 100$, $P(12,10) = 90$ and $P(0,16) = 48$. So make 20 bowls and no vases….unless someone really wants a vase.